The Search for Catchment-Scale Physics:
Spatial Scaling & Similarity with Hydrological Heterogeneity

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The Challenges

1. “... one could attempt to find simple equilibrium laws at the macroscale in much the same way as in the statistical mechanics approach”

2. “Unfortunately, in hydrology we have not established any principle of similarity for catchment behavior ...

and thus are in the situation that pertained in hydraulics 100 years ago before the introduction of the Reynolds number and the Froude number.”
Definitions (Blöschl & Sivapalan, HP 1995)

• **Scale** - characteristic length (or time) of process, observation, model
  • **Scaling** - transfer of information across scales

• **Similarity is present when …**
  • characteristics of one system can be related to the corresponding characteristics of another system by a simple conversion factor, called the scale factor
  • Examples:
    • ratio of catchment areas for relating flows of two catchments
    • \( \ln(a/\tan b) \) for relating depths to water table of locations
  • NB only exact under very strong assumptions!
Definitions (Blöschl & Sivapalan, HP 1995)

- Heterogeneity (or variability) – at multiple scales
Responses to these Challenges

1. REA (Representative Elementary Area) proposed as the fundamental scale for catchment modelling
2. Hydrologic Similarity explicitly includes runoff generation processes ("what to route" is what matters!)

Wood et al 1990, Reviews of Geophysics
1. Sampling Runoff over Scales

TOPMODEL

runoff

Many different catchment sizes

Effects Of Spatial Variability and Scale With Implications To Hydrologic Modeling
Wood et al, 1988, J.Hydrol, Eric’s most highly cited 1st author paper
REASuggests: it’s ok to use simpler models at scales > ~1km²

But, but, but...

Nature is more complicated than this (Woods et al, 1995; Seyfried and Wilcox, 1995)

REA is case-specific (Blöschl et al 1995)

Many, many, questions (Fan & Bras, 1995)

Luis will say more about REA implications
REW – Representative Elementary Watershed

• Formulate REW-scale dynamics (Reggiani et al, AWR, 1998)

• Need closure equations for fluxes at REW-scale (Beven, HP 2006)
  • “Holy Grail”, experimental data are crucial – progress so far?
What Else Can We Try?

- **Catchment response = “integration” over a distribution of parameters**

- **How does simple catchment-scale response emerge from heterogeneity?**

- **A1.** Catchments do integrate in space and time

- **A2.** Emergent processes

- **A3.** Perhaps also because parameters have the same distribution in different catchments?

All catchments have effectively the same topographic index distribution (Woods & Sivapalan, WRR 1997) Does this work for other parameters and their cross-correlations?
Current Approaches

**Upscaled Point Equations**
- Assumption that “point” scale equations can “work” at scales of tens and hundreds of metres
- Challenge to find the effective parameter values, given the constraints of computing resources

**“Macroscale” Equations**
- Variety of (conceptual) model structures as competing hypotheses
- Accept structures that are consistent with observations
- Challenges: obtain relevant observations; weak theoretical basis for subsurface sub-models

Arguments can be made for either approach. Should we do both? Why? How?
2. Similarity – Wood, Hebson, Sivapalan, Beven

• Series of 3 WRR papers (1986, 1987, 1990)
• Prior similarity analyses for flood response by Henderson & Wooding, Eagleson, Rodriguez-Iturbe, …,
  • Very simple runoff generation, focus on the routing processes
• “In this sequence of papers our aim is to provide a greater understanding of the interrelationships that underlie the storm response of catchments of different scales and physical characteristics by focusing on concepts of similarity” (Sivapalan et al, WRR 1987)
  • Flood frequency, storm response
  • Identified and linked together key controlling variables such as
    • dimensionless soil-topographic index (saturation excess)
    • ratio of rainfall to saturated soil hydraulic conductivity (infiltration excess)
    • spatial CV of catchment rain (ratio of storm area to catchment area)
    • ratio of storm duration to travel time (routing)
Similarity - Implications

• Potentially a powerful approach to making meaningful approximate statements about hydrological functioning

• Think in terms of relative amounts (fluxes, stores) for all hydrological process components

• A potential basis for catchment classification

• Testable method to transfer established knowledge
  • Across space to ungauged catchments
  • Across time scales in a catchment (e.g. climate → flood frequency)
  • Across space scales (e.g. spatial downscaling)

• We have been better at developing ideas than testing them
  • So we’re missing the opportunity to have ideas fail, and learn something new!
### Theoretical Promise of Process Representation

<table>
<thead>
<tr>
<th>Dimensionless groups</th>
<th>Dimensionless number</th>
<th>Interpretation (^1) (ratios)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\rho V l}{\mu} )</td>
<td>Reynolds number, Re</td>
<td>Inertia force to viscous force</td>
</tr>
<tr>
<td>( \frac{V}{\sqrt{g l}} )</td>
<td>Froude number, Fr</td>
<td>Inertia force to gravitational force</td>
</tr>
<tr>
<td>( \frac{p}{\rho V^2} )</td>
<td>Euler number, Eu</td>
<td>Pressure force to inertia force</td>
</tr>
</tbody>
</table>

#### Dimensionless groups | Dimensionless number
---|---
Climate | \( E_{r}/P \) | Aridity index, \( R \)
| \( |\delta_{p} - R \delta_{g}| \) | Seasonality index, \( S \)
Canopy and soil | \( w_{o}/(P/N) \) | Canopy storage index, \( W_c \)
| \( K/(P/N) \) | Relative infiltration, \( K \)
| \( w_{r}P_{s} \) | Rootzone storage index, \( W_r \)
Saturated flow | \( DL(T_0\tan\beta_{r}) \) | Advection response index, \( t_0 \)
| \( T_0\tan\beta/LP \) | Relative transmissivity, \( T_0 \)
| | | Slope of topographic index distribution, \( \omega \)

Wagener et al. 2007 Geography Compass
Similarity – Examples at Other Time Scales

Recession: Lyon & Troch 2007 WRR

Season: Laio et al 2001, AWR

Average runoff: Milly, 1994 WRR
Here are 3 independent, relevant indices. Use combinations of the 3 to define classes.

Application

(Berghuijs et al WRR 2014)
Catchments in the same class have similar hydrological signatures so:
1. It makes sense to analyse class members as a group
2. These indices are (uncalibrated, a priori) broad-scale predictors of hydrological similarity

This looks good but it is only a first step
• A theory-led empirical association between climate and hydrology
• Separates nature into groups, but doesn’t predict each group’s response
• What extra information do we need for this?
Similarity – Failure Can Be Good!

• Similarity predicts that catchment responses will be related by specific scaling factors
• Deviations from the scaling are a sign of incomplete knowledge
• Consistent deviations can be informative!

Snow-dominated catchments produce more runoff
(NB: this picture is merely suggestive)
Berghuijs et al (2014 Nature Climate Change)
Similarity – Further Progress

• Still a major gap between the event/recession/within-season time scale and the annual/longer scales, with notable exceptions:
  • Laio et al (2002, JGR)
  • Sivapalan et al (2005 WRR)
  • Iacobellis/Manfreda/Fiorentino

• This gap manifests as
  • Limited theoretical connection across time scales
  • No consistency in treatment of spatial heterogeneity with scale
  • Limited treatment of transport processes across scales (though that is changing)
    • Lack of practical utility for similarity indices at timescales less than annual

• Also gaps for connectivity; groundwater-dominated systems

• We are still waiting for the GUTH - Grand Unified Theory of Hydrology
Summary

• REA: proposed as fundamental scale for catchment modelling
  • The REA inspired the search for a sound linkage between point-scale and catchment scale representations of hydrology
  • Still looking for soundly-based catchment-scale physics

• Hydrologic similarity gives us simple ways to understand hydrology
  • There are opportunities to link across time scales and space scales
  • There are opportunities to learn through real-world testing, especially on large, challenging, diverse datasets
  • Similarity is extremely valuable for generating better ways to classify